Software Performance Engineering

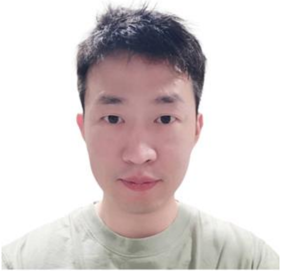
Course Website

software-performance-engineering.github.io/

Xuhao Chen

Tuesday, August 26, 2025

Logistics




- **Lecturer : Xuhao Chen**
- **TA : Sophia Sun**
- **Refrain from using laptops during the lectures**
- **Course site:** software-performance-engineering.github.io/
- Questions? **Piazza**
  - Read **Course Info** handout on the site thoroughly
  - **HW0** on Gradescope — due tonight 10pm!
  - **Recitation** (**mandatory**) mostly on **Tuesdays** (bring laptop)
  - Finish **HW1 Checkoff** ASAP — due Friday 10pm!
  - Read **Project-1** handout before the first recitation

Software Performance Engineering

LECTURE 1

Introduction & Matrix Multiplication

Xuhao Chen

Tuesday, August 26, 2025

Why Software Performance Engineering (SPE)?

Is Performance Important?

What software properties are more important than performance?

- Functionality
- Correctness
- Security

Is Performance Important?

What software properties are more important than performance?

- Compatibility
- Correctness
- Clarity
- Debuggability
- Functionality
- Maintainability
- Modularity
- Portability
- Reliability
- Robustness
- Security
- Usability

⋯ and more.

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⋯ and more.

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Choosies

Choose one of two objects to take back to your seat.

or

Object 1

Object 2

Adam Smith's Paradox

Adam Smith
1723-1790

"The word ***VALUE***, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called "value in use;" the other, "value in exchange." The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water; but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it." — *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776)

Is Performance Important?

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··· and more.

If these properties are more important, then **why study performance?**

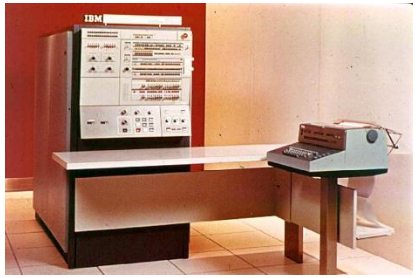
Performance is the **currency** of computing. You can often "buy" needed properties with performance

A Brief History of Performance Engineering

Computer Programming in the Early Days

Long ago, software performance engineering (**SPE**) was **common**, because **machine resources were limited.**

IBM System/360



Launched:	1964
Clock rate:	33 KHz
Data path:	32 bits
Memory:	524 Kbytes
Cost:	$250,000

DEC PDP-11

Launched:	1970
Clock rate:	1.25 MHz
Data path:	16 bits
Memory:	56 Kbytes
Cost:	$20,000

Apple II

Launched:	1977
Clock rate:	1 MHz
Data path:	8 bits
Memory:	48 Kbytes
Cost:	$1,395

Many applications strained machine resources.

- Programs had to be planned around the machine.
- Many programs would not “fit” without intense performance engineering.

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Long ago, software performance engineering (**SPE**) was **common**, because **machine resources were limited.**

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Clock rate:	33 KHz	1.25 MHz	1 MHz
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- Programs had to be planned around the machine.
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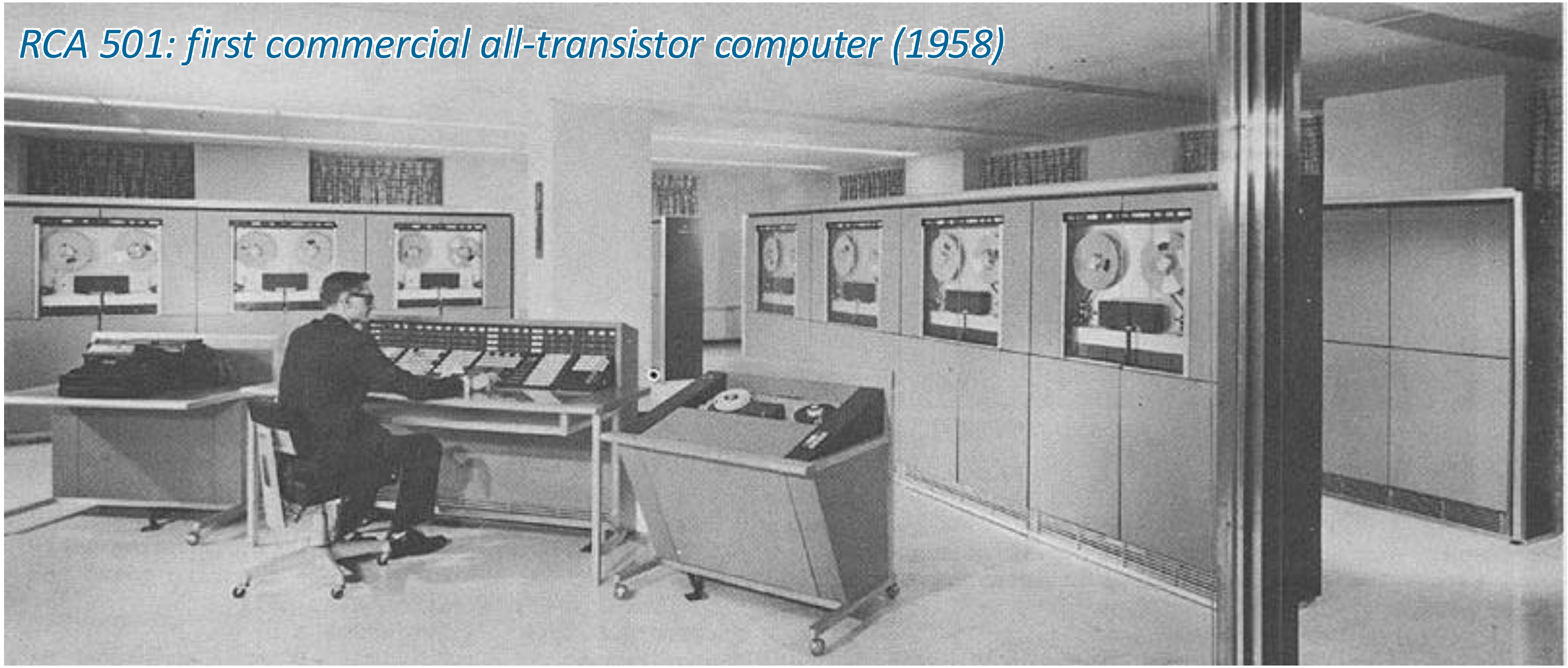
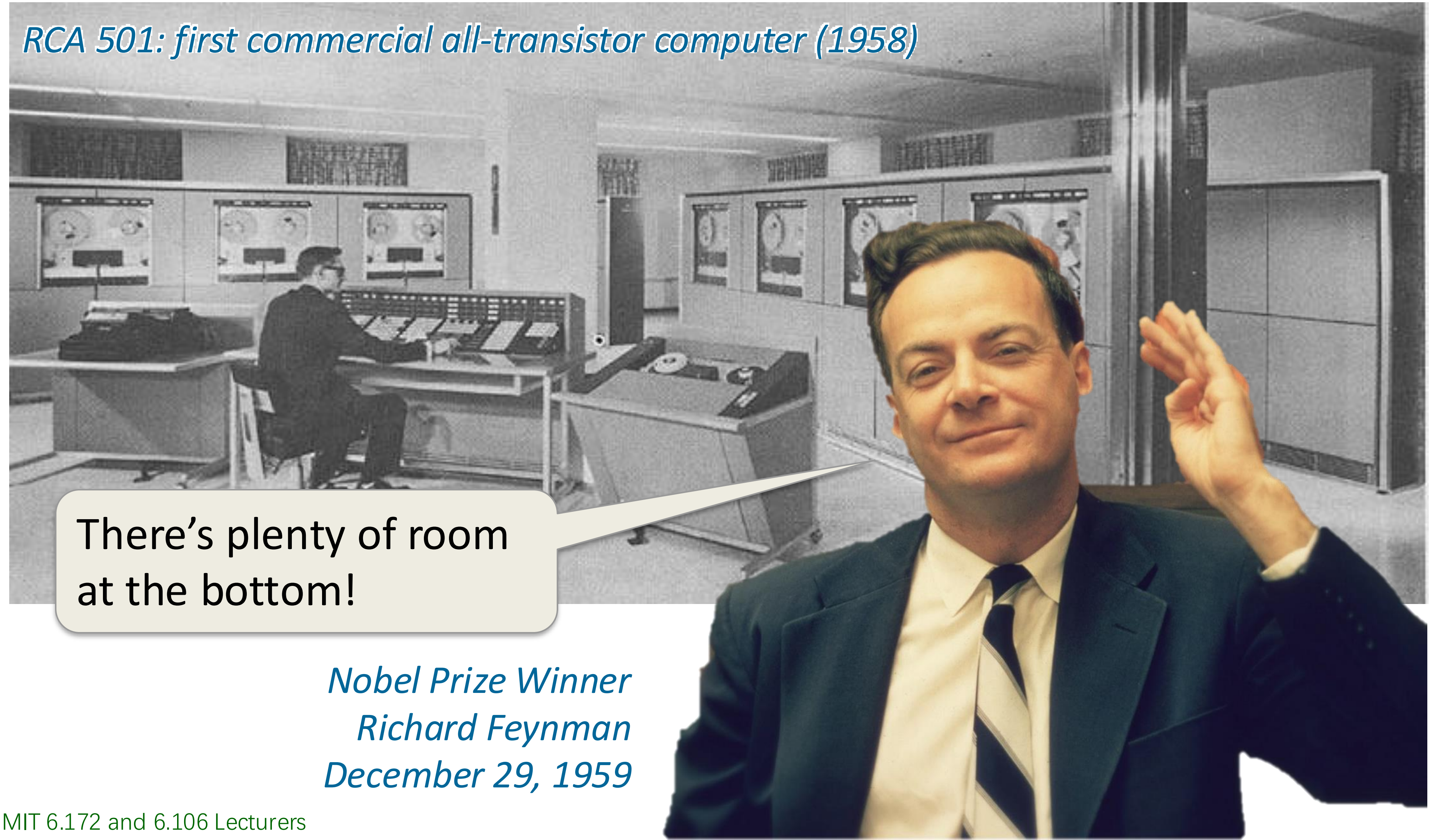
SOFTWARE PERFORMANCE ENGINEERING RULED!

The Early Days of Computing



The Early Days of Computing

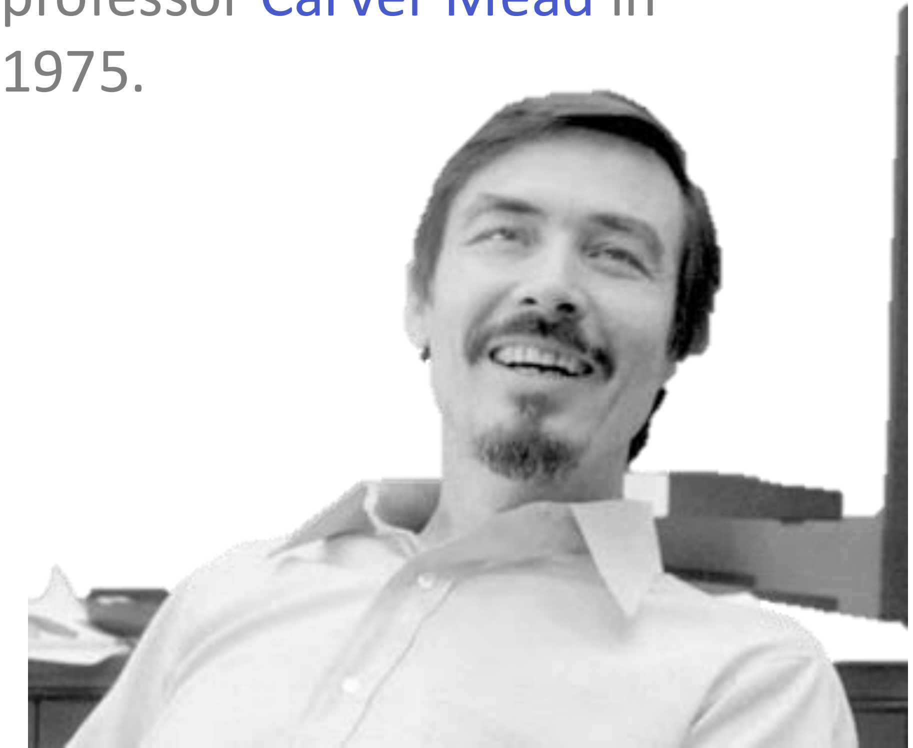


Nobel Prize Winner
Richard Feynman
December 29, 1959

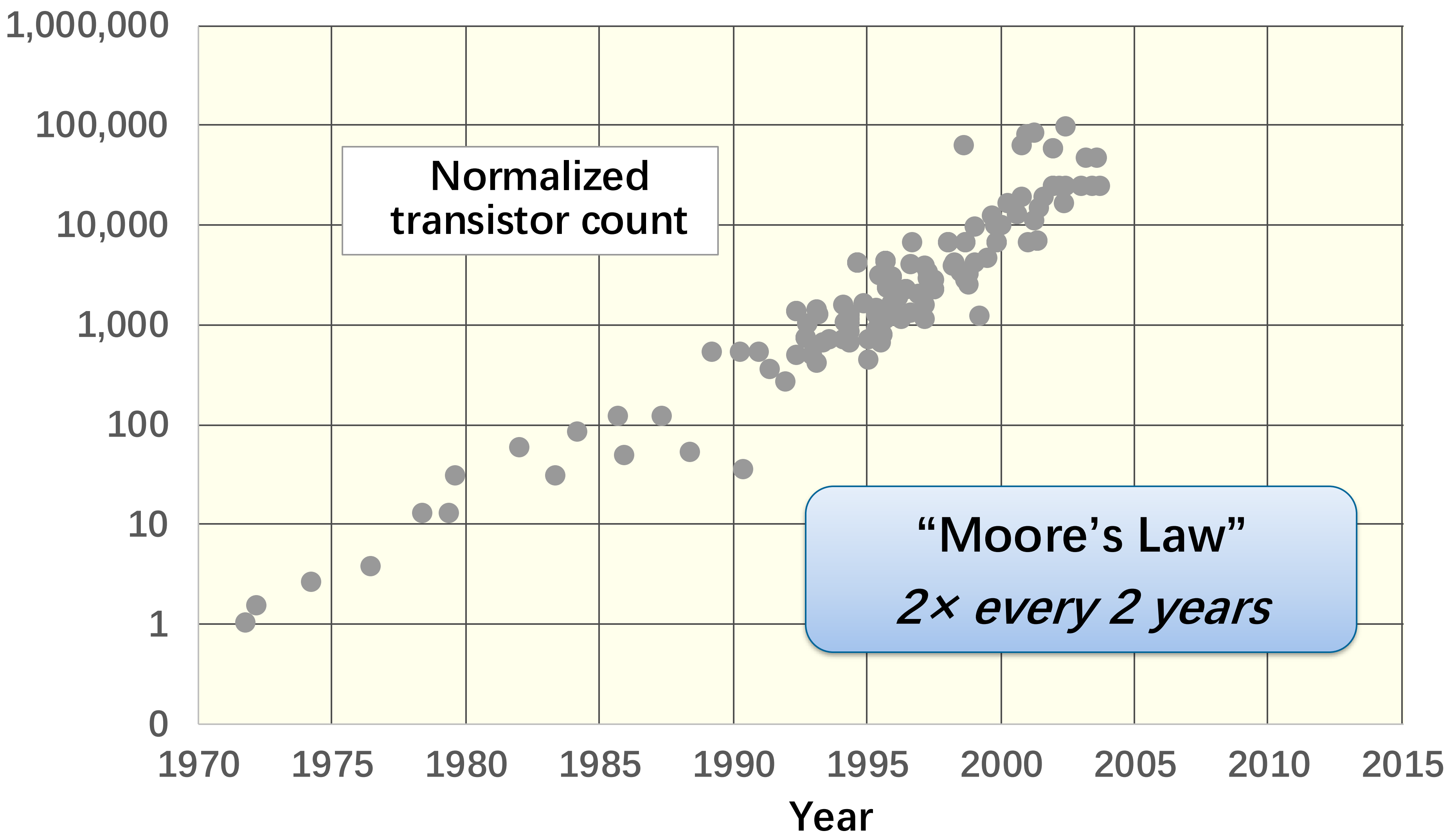
MOORE'S LAW

MOORE'S LAW is an economic and technology trend originally articulated in 1965 by Intel founder Gordon Moore.

The trend was christened "MOORE'S LAW" by Caltech professor Carver Mead in 1975.

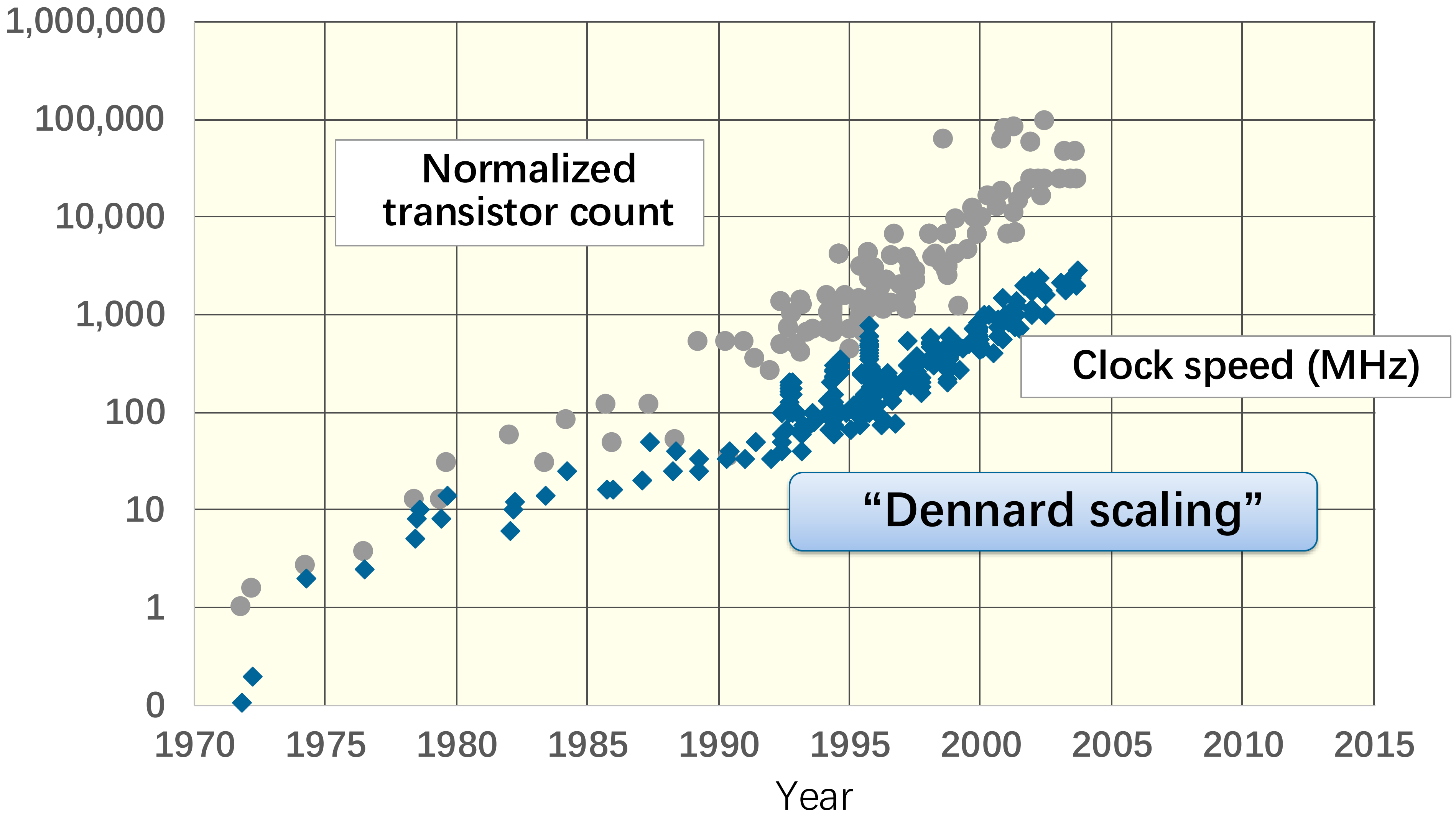


Technology Scaling from 70's to 2004



Processor data from Stanford's CPU DB [DKM12].

Technology Scaling from 70's to 2004



Processor data from Stanford's CPU DB [DKM12].

Advances in Hardware

Apple computers with similar prices from 1977 to 2004

Apple II

Launched:	1977
Clock rate:	1 MHz
Data path:	8 bits
Memory:	48 KB
Cost:	$1,395

Power Macintosh G4

Launched:	2000
Clock rate:	400 MHz
Data path:	32 bits
Memory:	64 MB
Cost:	$1,599

Power Macintosh G5

Launched:	2004
Clock rate:	1.8 GHz
Data path:	64 bits
Memory:	256 MB
Cost:	$1,499

Advances in Hardware

Apple computers with similar prices from 1977 to 2004

	Apple II	Power Macintosh G4	Power Macintosh G5
Launched:	1977	2000	2004
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SOFTWARE
PERFORMANCE
ENGINEERING
RULED!

Advances in Hardware

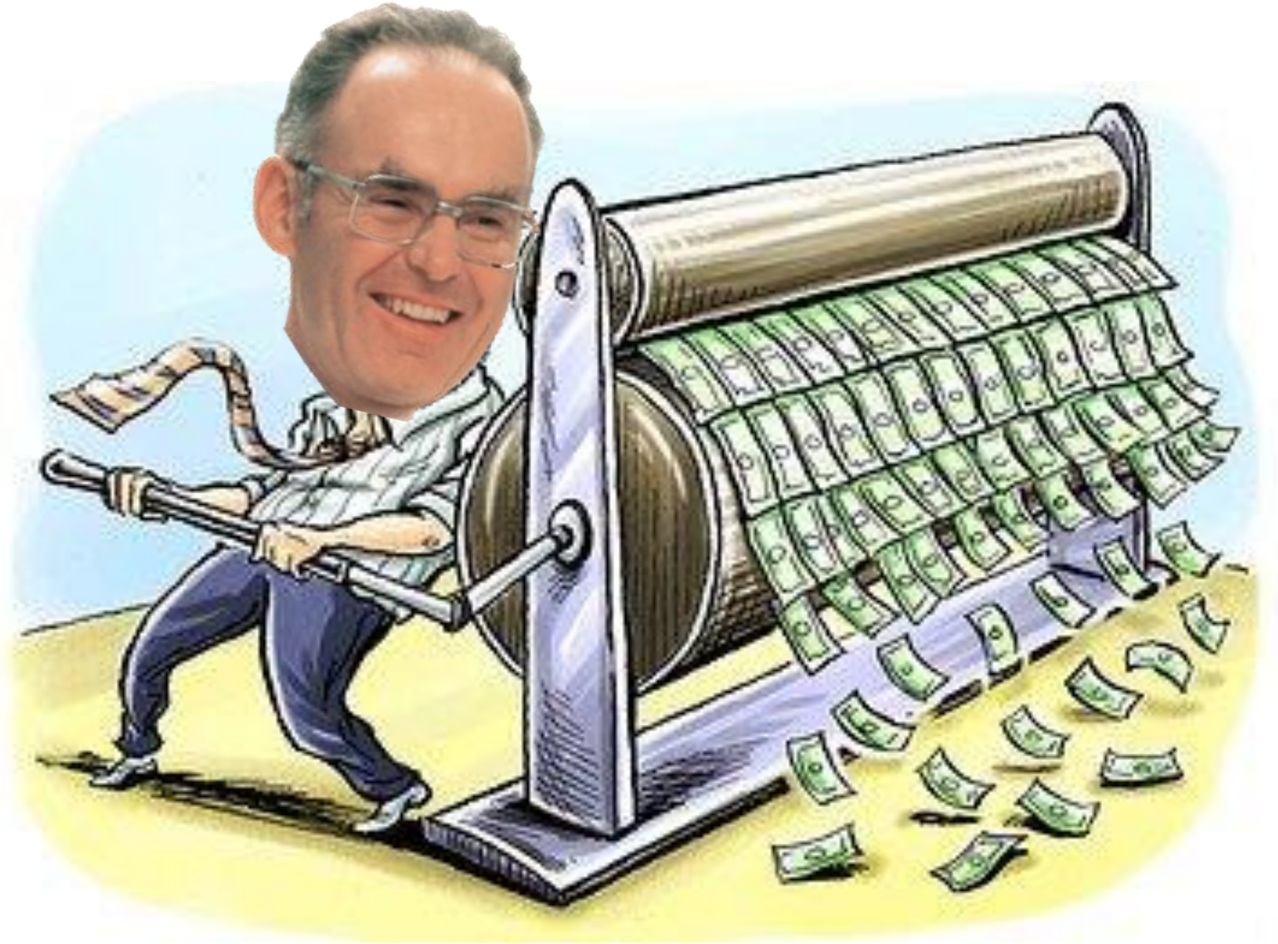
Apple computers with similar prices from 1977 to 2004

HARDWARE PERFORMANCE RULED!

Apple II		Power Macintosh G4		Power Macintosh G5	
Launched:	1977	Launched:	2000	Launched:	2004
Clock rate:	1 MHz	Clock rate:	400 MHz	Clock rate:	1.8 GHz
Data path:	8 bits	Data path:	32 bits	Data path:	64 bits
Memory:	48 KB	Memory:	64 MB	Memory:	256 MB
Cost:	$1,395	Cost:	$1,599	Cost:	$1,499

Until 2004

Moore's Law and the scaling of clock frequency
= printing press for the currency of performance.



Lessons Learned in the Beginning of this Era

More computing sins are committed in the name of efficiency (without necessarily achieving it) than for any other single reason — including blind stupidity. [W79]

William A. Wulf

Lessons Learned in the Beginning of this Era

The First Rule of Program Optimization: Don't do it.
The Second Rule of Program Optimization — For experts only: Don't do it yet. [J88]

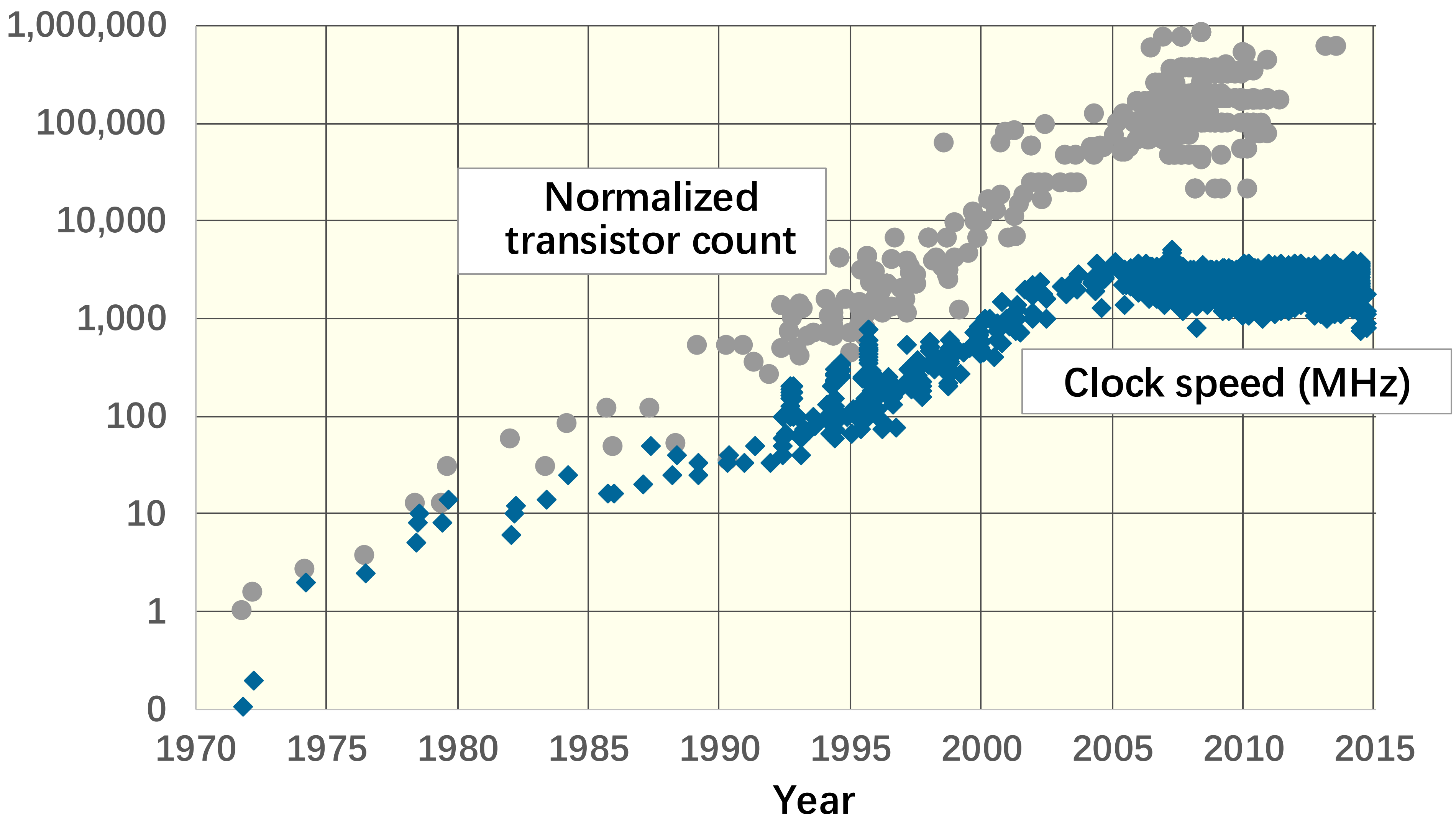
Michael A. Jackson

Lessons Learned in the Beginning of this Era

Premature optimization is the root of all evil. [K79]

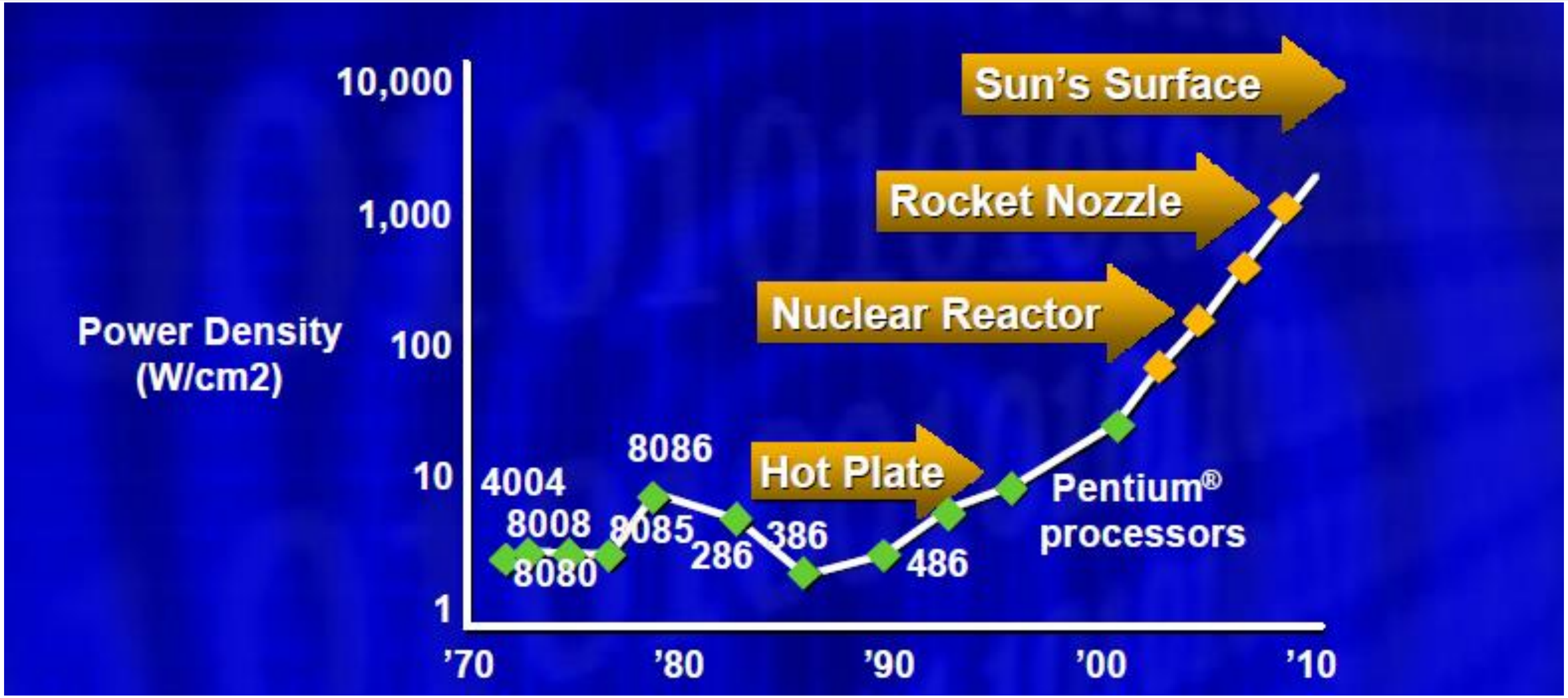
Donald E. Knuth

Technology Scaling After 2004



Processor data from Stanford's CPU DB [DKM12].

Power Density

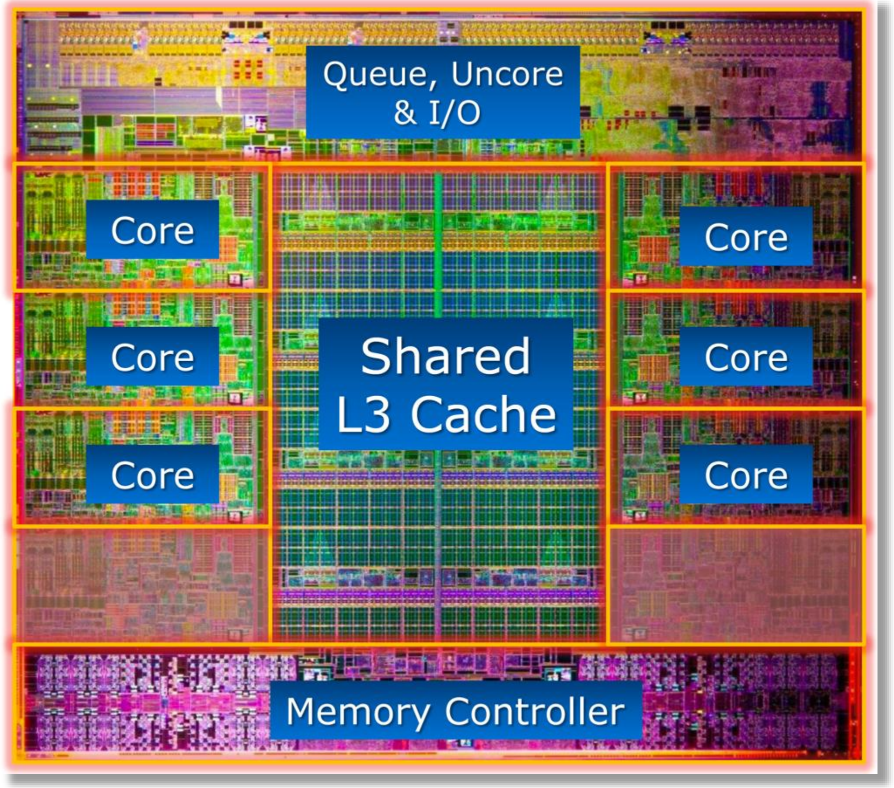


Source: Patrick Gelsinger, *Intel Developer's Forum*, Intel Corporation, 2004.

The growth of power density, as seen in 2004, if the scaling of clock frequency had continued its trend of 25%-30% increase per year.

Vendor Solution: Multicore

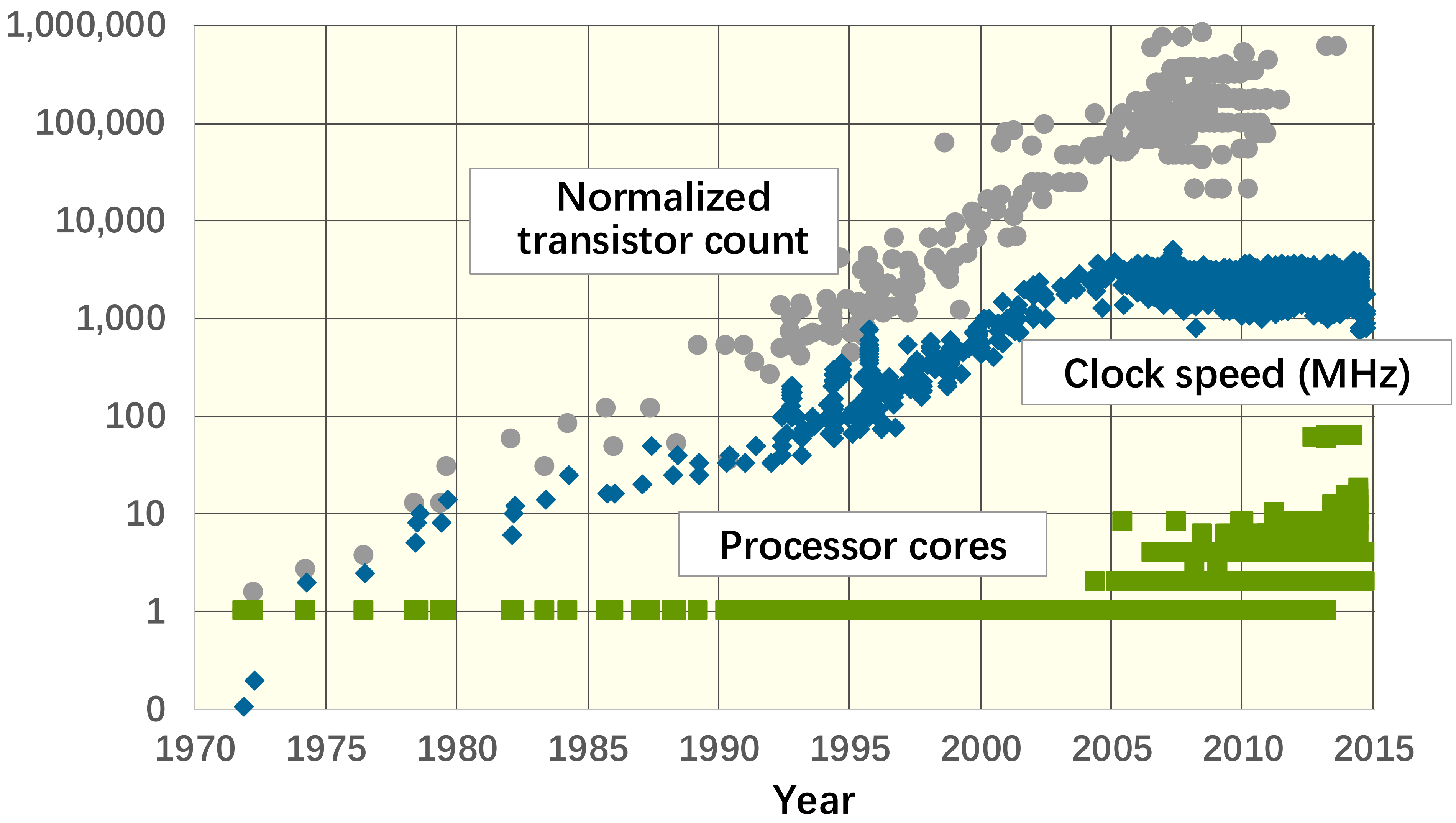
- To scale performance, vendors put many processing cores on the chip
- Each generation of Moore's Law potentially doubles the number of cores



Intel Core i7 3960X
(Sandy Bridge E), 2011

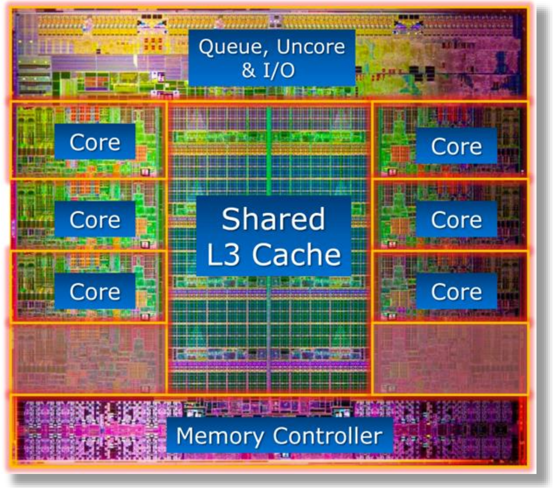
- 6 cores
- 3.3 GHz
- 15-MB L3 cache

Technology Scaling

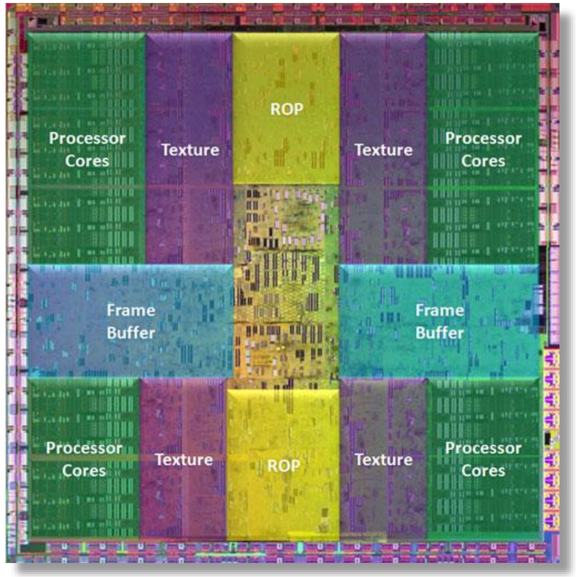


Processor data from Stanford's CPU DB [DKM12].

Performance Is No Longer Free



2011 Intel Skylake processor



2008 NVIDIA GT200 GPU

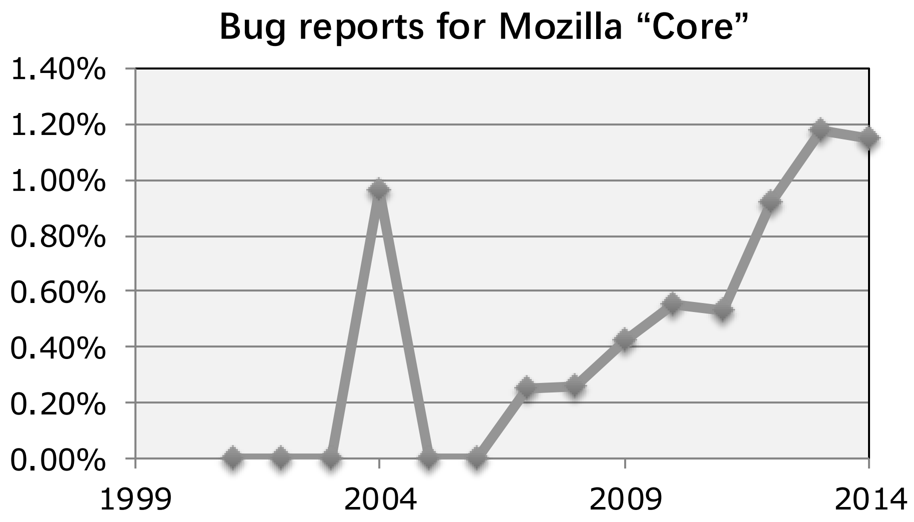
Moore's Law continued to increase computer performance.

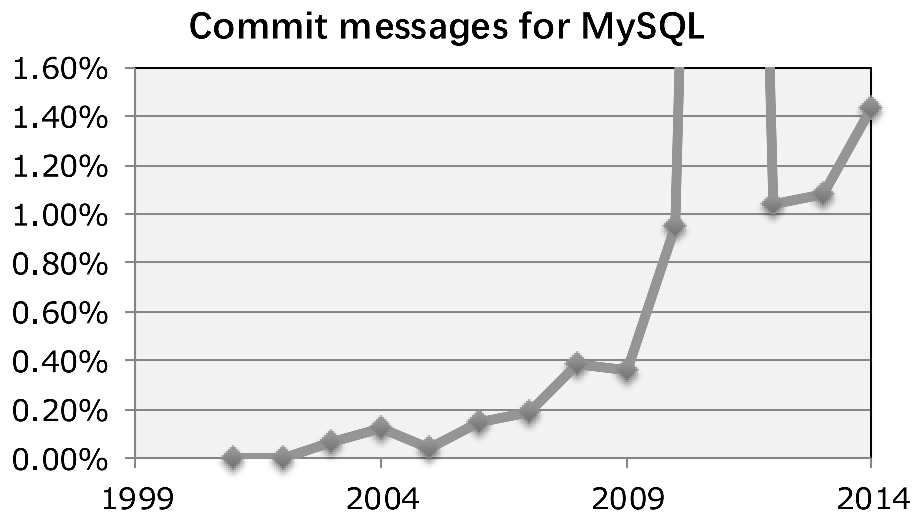
But now that performance was available in the form of **multicores**

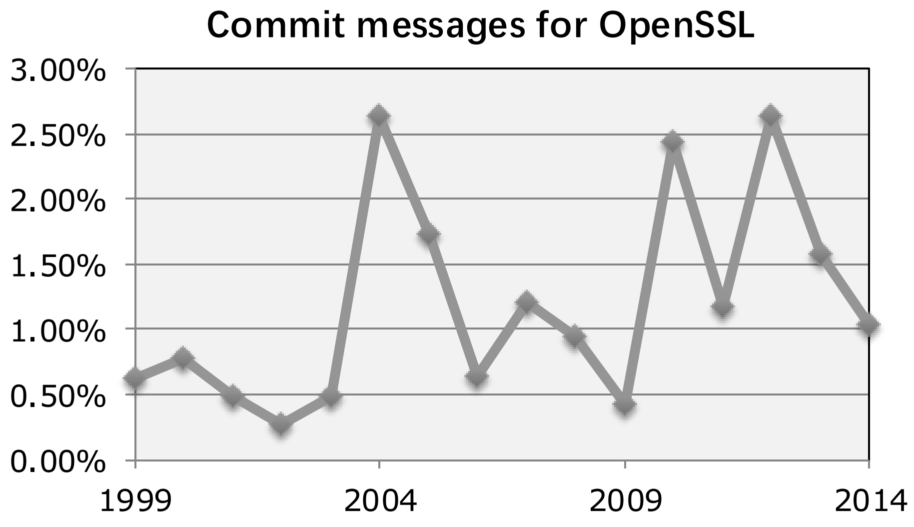
with complex caches, vector units, GPU's, FPGA's, etc.

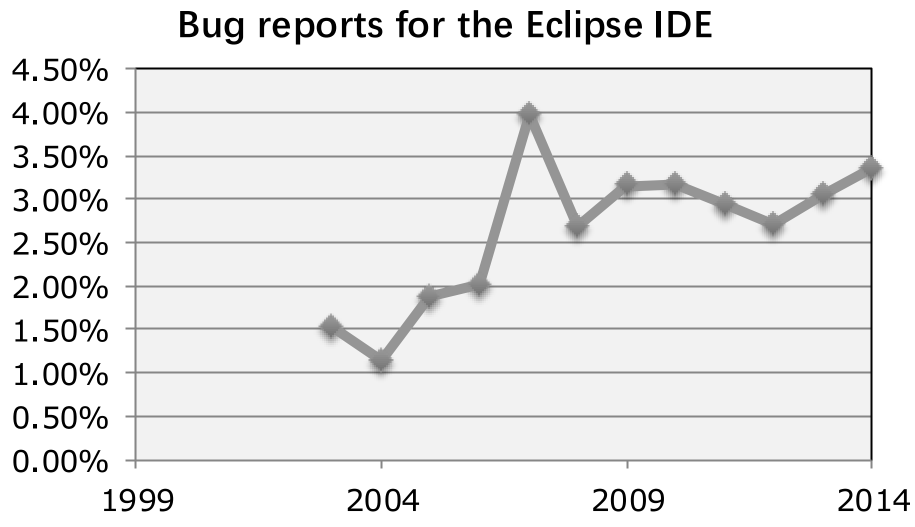
Software must be **adapted** to utilize the hardware efficiently!

Software Bugs Mentioning "Performance"

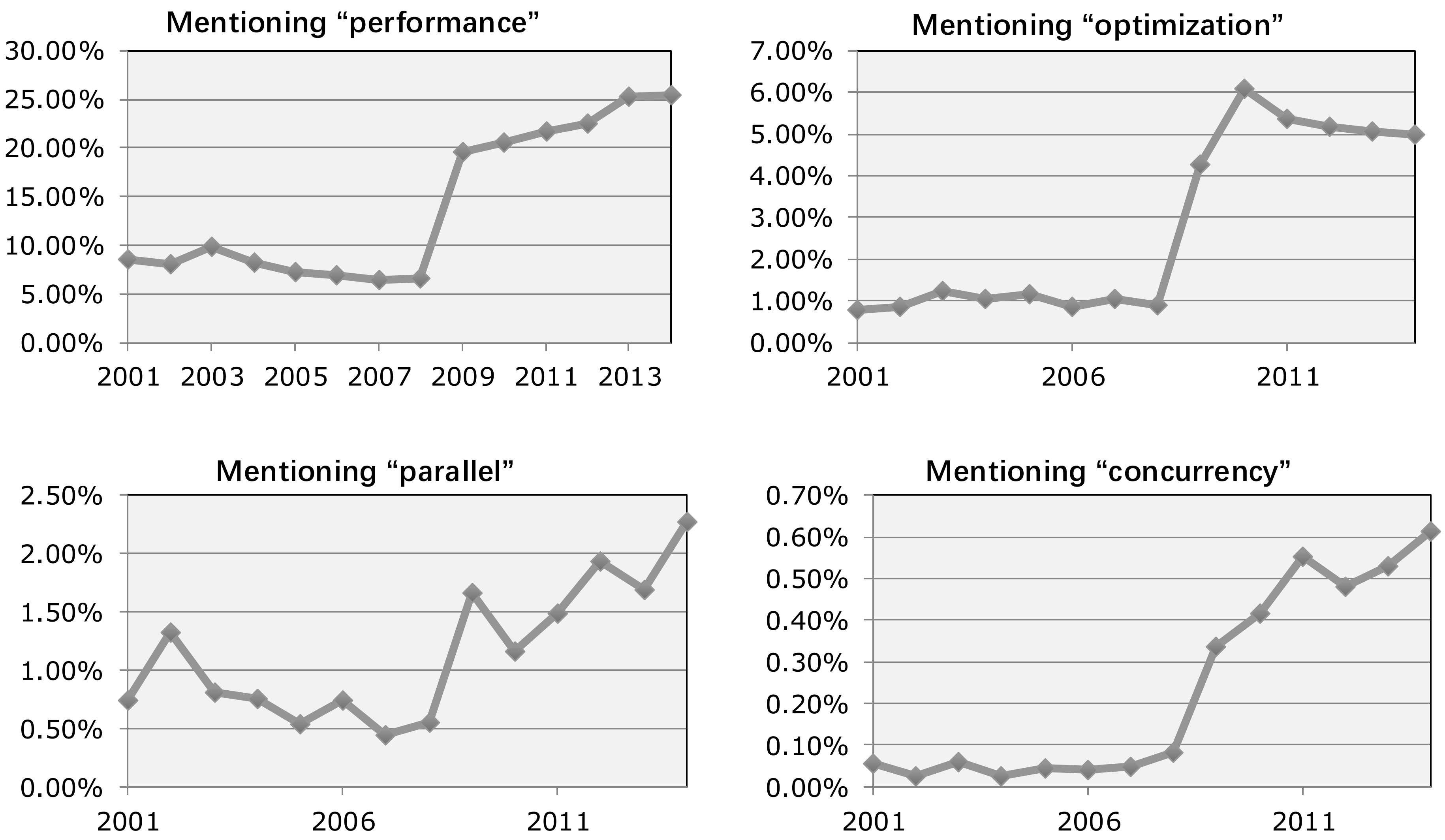








Software Developer Jobs



Source: Monster.com

And Now, Moore's Law Is Over!

Where Are We Now?

- Intel achieved 14 nanometers in 2014
- According to Moore's Law, Intel should have achieved
  - 10 nanometers in 2016,
  - 7 nanometers in 2018,
  - 5 nanometers in 2020.
- But Intel did not release 10 nanometers until 2019!
- It took 5 years for what historically had taken only 2 years

Semiconductor technology will no longer give applications free performance.

Why Must the Party End?

Darn That Physics!

- It's implausible that semiconductor technologists can make **wires thinner than atoms**, which are at most a few angstroms across.
- The silicon lattice constant is 0.543 nanometers = 5.43 angstroms.

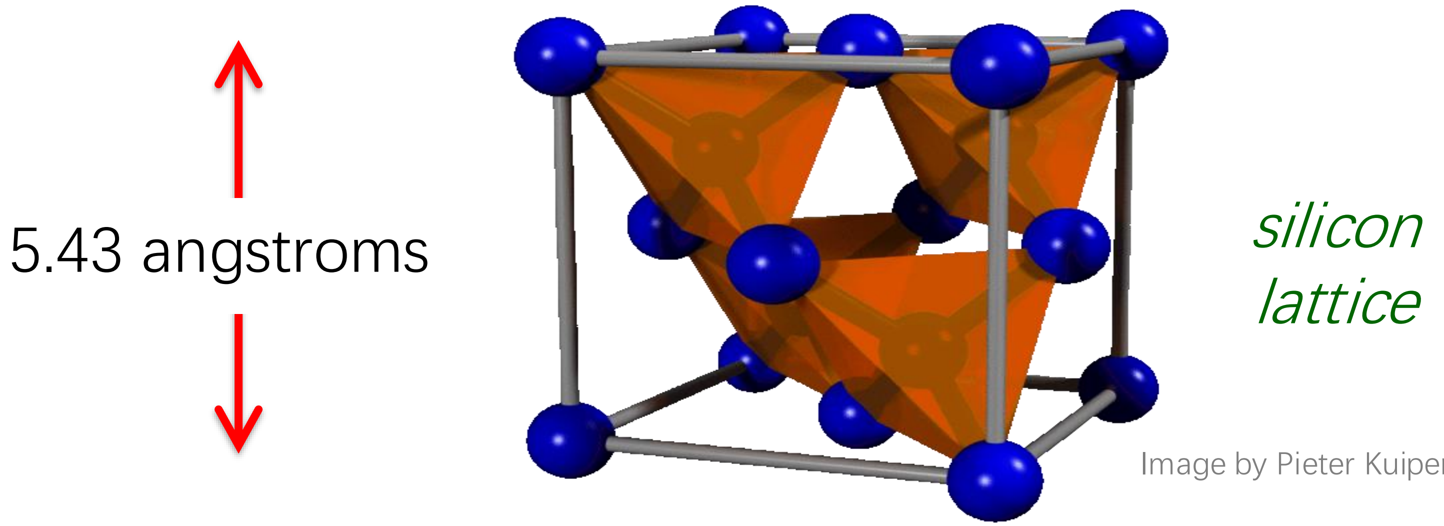


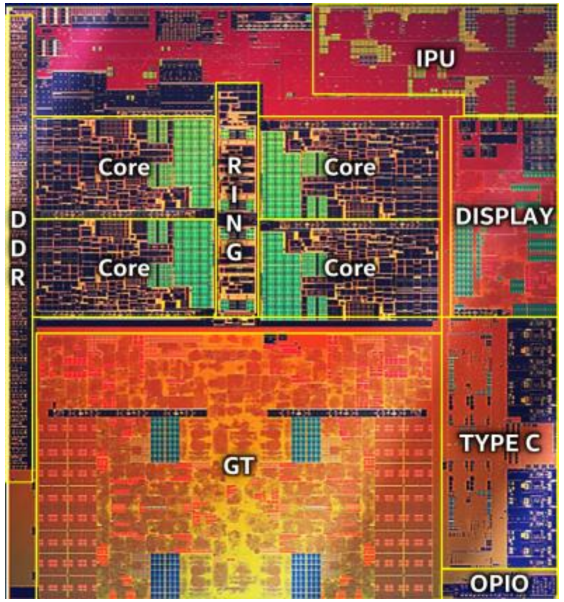
Image by Pieter Kuiper, Wikipedia Commons.

- **Technology roadmaps** see an end to transistor scaling around 5 nanometers. We're almost there!

The Printing Press Is Grinding to a Halt

Performance Engineering Redux

- A modern multicore desktop processor contains
  - parallel-processing cores
  - vector units
  - caches
  - instruction prefetchers
  - GPU's
  - hyperthreading
  - dynamic frequency scaling
  - …



2019 Intel 10nm processor

- These features can be challenging to exploit

In this class you will learn the principles and practice of writing fast code.

Case Study
Matrix Multiplication

Square-Matrix Multiplication

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$$

C A B

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Assume for simplicity that $n = 2^k$.

AWS c4.8xlarge Machine Specs

Feature	Specification
Microarchitecture	Haswell (Intel Xeon E5-2666 v3)
Clock frequency	2.9 GHz
Processor chips	2
Processing cores	9 per processor chip
Hyperthreading	2 way
Floating-point unit	8 double-precision operations, including fused-multiply-add, per core per cycle
Cache-line size	64 B
L1-icache	32 KB private 8-way set associative
L1-dcache	32 KB private 8-way set associative
L2-cache	256 KB private 8-way set associative
L3-cache (LLC)	25 MB shared 20-way set associative
DRAM	60 GB

Peak $= (2.9 \times 10^9) \times 2 \times 9 \times 16 = 836$ GFLOPS

Version 1: Nested Loops in Python

```
import sys, random
from time import *

n = 4096

A = [[random.random()
      for row in xrange(n)]
     for col in xrange(n)]
B = [[random.random()
      for row in xrange(n)]
     for col in xrange(n)]
C = [[0 for row in xrange(n)]
     for col in xrange(n)]

start = time()
for i in xrange(n):
    for j in xrange(n):
        for k in xrange(n):
            C[i][j] += A[i][k] * B[k][j]
end = time()

print '%0.6f' % (end - start)
```

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print '%0.6f' % (end - start)
```

Running time:

≈ 6 microseconds?

≈ 6 milliseconds?

≈ 6 seconds?

≈ 6 hours?

≈ 6 days?

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end = time()

print '%0.6f' % (end - start)
```

Running time:
= 21042 seconds
≈ 6 hours

Is this fast?

Should we expect more from our machine?

Version 1: Nested Loops in Python

```python
import sys, random
from time import *

n = 4096

A = [[random.random()
      for row in xrange(n)]
     for col in xrange(n)]

B = 


C = 


star
for




end

print '%0.6f' % (end - start)
```

Running time
= 21042 seconds
≈ 6 hours

Is this fast?

Back-of-the-envelope calculation

$2n^3 = 2(2^{12})^3 = 2^{37}$ floating-point operations
Running time = 21042 seconds
∴ Python gets $2^{37}/21042 \approx 6.25$ MFLOPS
Peak ≈ 836 GFLOPS
Python gets ≈ 0.00075% of peak

Version 2: Java

```
import java.util.Random;

public class mm_java {
  static int n = 4096;
  static double[][] A = new double[n][n];
  static double[][] B = new double[n][n];
  static double[][] C = new double[n][n];

  public static void main(String[] args) {
    Random r = new Random();

    for (int i=0; i<n; i++) {
      for (int j=0; j<n; j++) {
        A[i][j] = r.nextDouble();
        B[i][j] = r.nextDouble();
        C[i][j] = 0;
      }
    }

    long start = System.nanoTime();

    for (int i=0; i<n; i++) {
      for (int j=0; j<n; j++) {
        for (int k=0; k<n; k++) {
          C[i][j] += A[i][k] * B[k][j];
        }
      }
    }

    long stop = System.nanoTime();

    double tdiff = (stop - start) * 1e-9;
    System.out.println(tdiff);
  }
}
```

Running time = 2,738 seconds
≈ 46 minutes

… about 8.8× faster than Python.


```
for (int i=0; i<n; i++) {
  for (int j=0; j<n; j++) {
    for (int k=0; k<n; k++) {
      C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Version 3: C

```c
#include <stdlib.h>
#include <stdio.h>
#include <sys/time.h>

#define n 4096
double A[n][n];
double B[n][n];
double C[n][n];

float tdiff(struct timeval *start,
            struct timeval *end) {
  return (end->tv_sec-start->tv_sec) +
    1e-6*(end->tv_usec-start->tv_usec);
}

int main(int argc, const char *argv[]) {
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      A[i][j] = (double)rand() / (double)RAND_MAX;
      B[i][j] = (double)rand() / (double)RAND_MAX;
      C[i][j] = 0;
    }
  }

  struct timeval start, end;
  gettimeofday(&start, NULL);

  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      for (int k = 0; k < n; ++k) {
        C[i][j] += A[i][k] * B[k][j];
      }
    }
  }

  gettimeofday(&end, NULL);
  printf("%0.6f\n", tdiff(&start, &end));
  return 0;
}
```

Using the Clang/LLVM 5.0 compiler

Running time = 1,156 seconds
≈ 19 minutes,

or about 2× faster than Java and about 18× faster than Python.

```c
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < n; ++j) {
    for (int k = 0; k < n; ++k) {
      C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Where We Stand So Far

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.007	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.119	0.014

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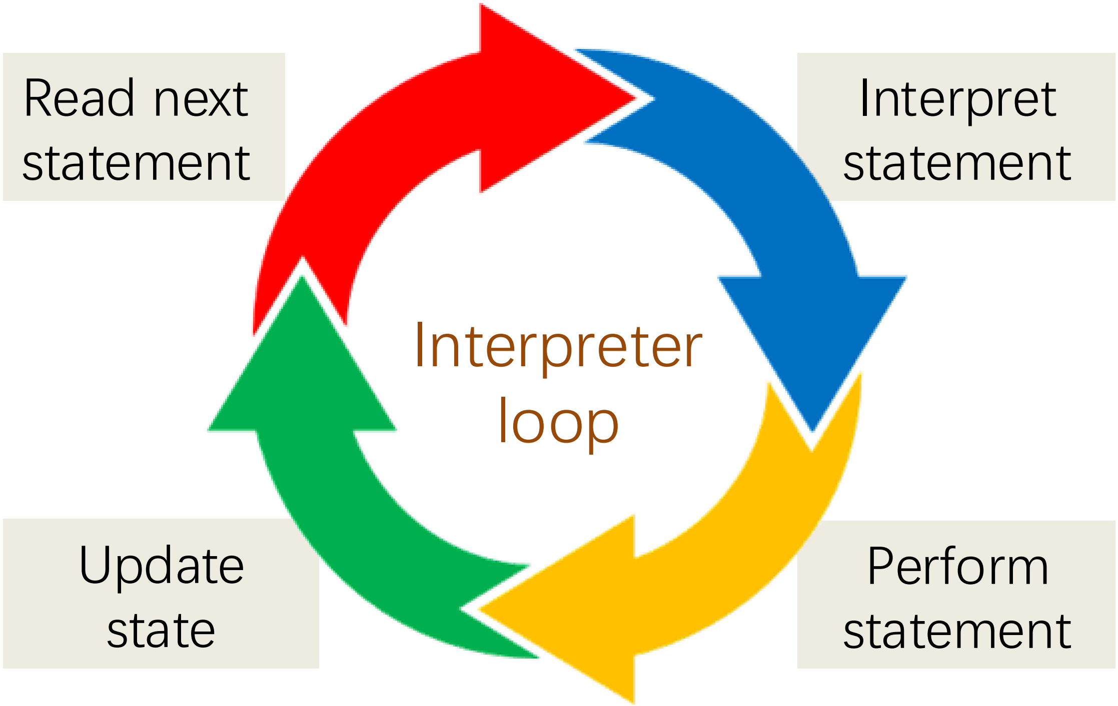
Why is Python so slow and C so fast?

- Python is interpreted.
- C is compiled directly to machine code.
- Java is compiled to byte-code, which is then interpreted and just-in-time (JIT) compiled to machine code.

Interpreters are versatile, but slow

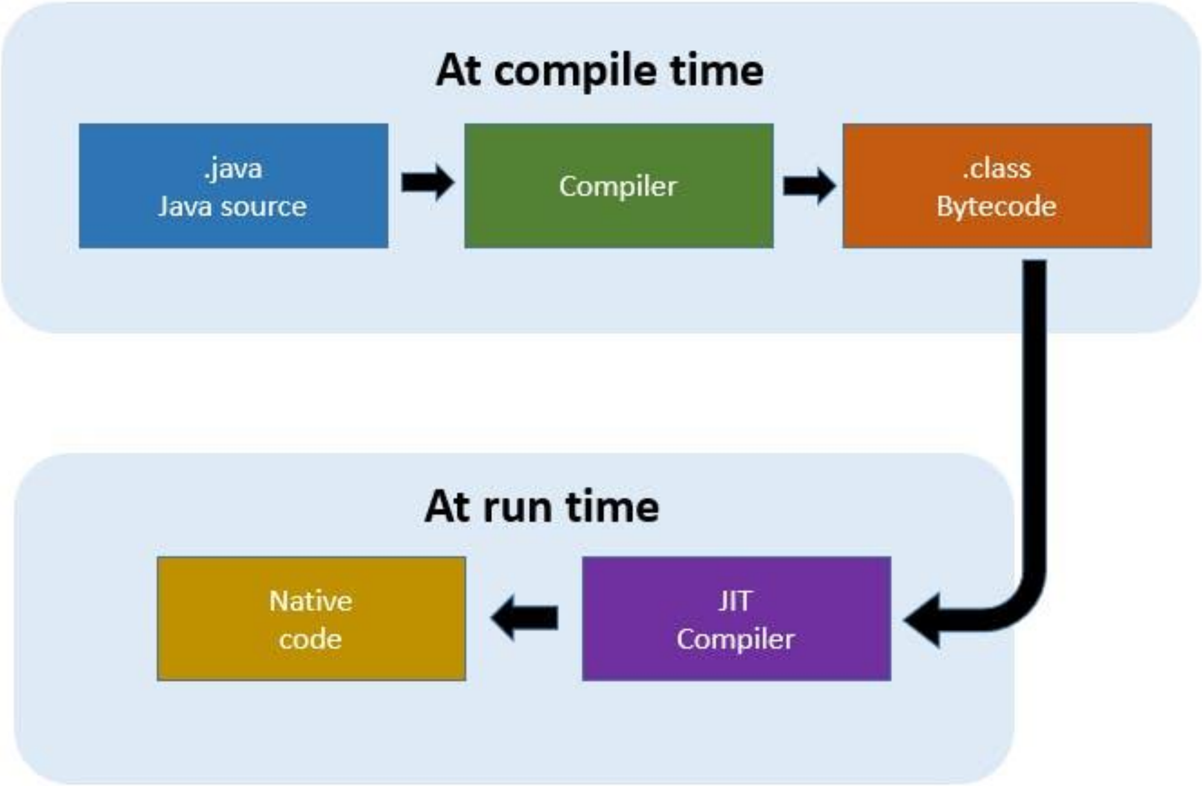
- The interpreter reads, interprets, and performs each program statement and updates the machine state.
- Interpreters can easily support high-level programming features — such as dynamic code alteration — at the cost of performance.



Just-In-Time Compilation in Java

JIT compilers can reduce some of the interpretation overhead

- When code is **first executed**, it is **interpreted**
- It identifies **hot code** that executes frequently
- Hot code gets **compiled** to machine code
- Future executions of that code use the more-efficient **compiled** version



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Loop Order

We can change the order of the loops in this program without affecting its correctness.

```
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < n; ++j) {
    for (int k = 0; k < n; ++k) {
      C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Loop Order

We can change the order of the loops in this program without affecting its correctness.

```
for (int i = 0; i < n; ++i) {
    for (int k = 0; k < n; ++k) {
  for (int j = 0; j < n; ++j) {
      C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Does the order of loops matter for performance?

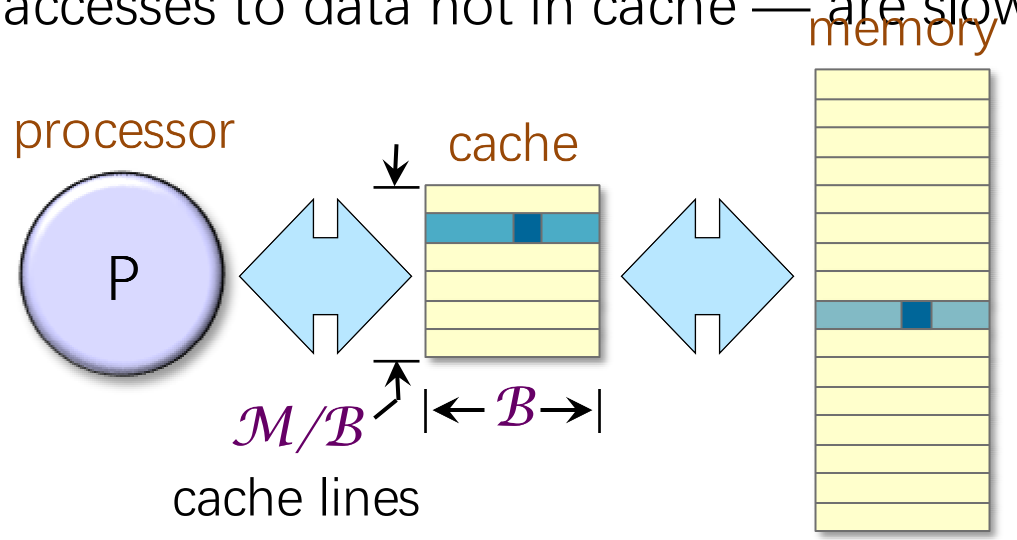
Performance of Different Orders

Loop order (outer to inner)	Running time (s)
`i, j, k`	1155.77
`i, k, j`	177.68
`j, i, k`	1080.61
`j, k, i`	3056.63
`k, i, j`	179.21
`k, j, i`	3032.82

- Loop order affects running time by a factor of **18**!
- What's going on?

Hardware Caches

- Each processor reads and writes main memory in contiguous blocks, called **cache lines**.
- Previously accessed cache lines are stored in a smaller memory, called a **cache**, that sits near the processor.
  - **Cache hits** — accesses to data in cache — are fast.
  - **Cache misses** — accesses to data not in cache — are slow.



Performance of Different Orders

We can measure the effect of cache using the **`cachegrind`** tool:

```
$ valgrind --tool=cachegrind ./mm
```

Loop order (outer to inner)	Running time (s)	Last-level-cache miss rate
i, j, k	1155.77	7.7%
i, k, j	177.68	1.0%
j, i, k	1080.61	8.6%
j, k, i	3056.63	15.4%
k, i, j	179.21	1.0%
k, j, i	3032.82	15.4%

Version 4: Interchange Loops

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093

Version 4: Interchange Loops

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093

What other simple changes we can try?

Compiler Optimization

- `clang` provides a collection of **optimization switches**
  - You can specify a switch to the compiler to ask it to optimize
- `clang` also supports optimization levels
  - Generally, **higher** optimization levels produce **faster** code

Opt. level	Meaning	Time (s)
`-O0`	Do not optimize	177.54
`-O1`	Optimize	66.24
`-O2`	Optimize even more	54.63
`-O3`	Optimize yet more	55.58

Version 5: Optimization Flags

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093
5	+ optimization flags	54.63	3.25	385	2.516	0.301

With simple code and compiler technology,
we can achieve **0.3%** of the peak performance of the machine.

Version 5: Optimization Flags

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
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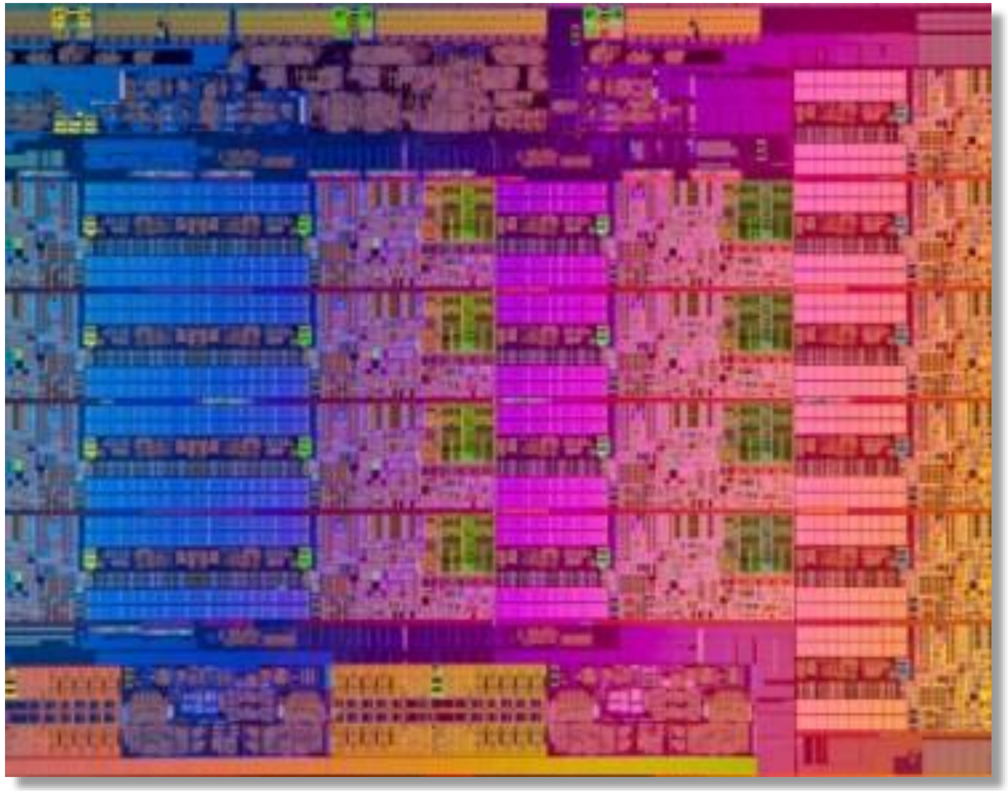
With simple code and compiler technology,
we can achieve **0.3%** of the peak performance of the machine.

Where can we get more performance?

Multicore Parallelism



Intel Haswell E5:
9 cores per chip

The AWS test machine has 2 of these chips.

We're running on just 1 of the 18 parallel-processing cores on this system. Let's use them all!

Parallel Loops

A `cilk_for` loop enables all iterations of the loop to execute in parallel.

```
cilk_for (int i = 0; i < n; ++i)
  for (int k = 0; k < n; ++k)
    cilk_for (int j = 0; j < n; ++j)
      C[i][j] += A[i][k] * B[k][j];
```

Both of these loops can be parallelized.

Which parallel version works best?

- parallelize just the `i` loop,
- parallelize just the `j` loop, or
- parallelize both the `i` and `j` loops.

Experimenting with Parallel Loops

Parallel `i` loop

```
cilk_for (int i = 0; i < n; ++i)
  for (int k = 0; k < n; ++k)
    for (int j = 0; j < n; ++j)
      C[i][j] += A[i][k] * B[k][j];
```

Running time: **3.18s**

Parallel `j` loop

```
for (int i = 0; i < n; ++i)
  for (int k = 0; k < n; ++k)
    cilk_for (int j = 0; j < n; ++j)
      C[i][j] += A[i][k] * B[k][j];
```

Running time: **531.71s**

Parallel `i` and `j` loops

```
cilk_for (int i = 0; i < n; ++i)
  for (int k = 0; k < n; ++k)
    cilk_for (int j = 0; j < n; ++j)
      C[i][j] += A[i][k] * B[k][j];
```

Running time: **10.64s**

Version 6: Parallel Loops

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093
5	+ optimization flags	54.63	3.25	385	2.516	0.301
6	Parallel loops	3.04	17.97	6,921	45.211	5.408

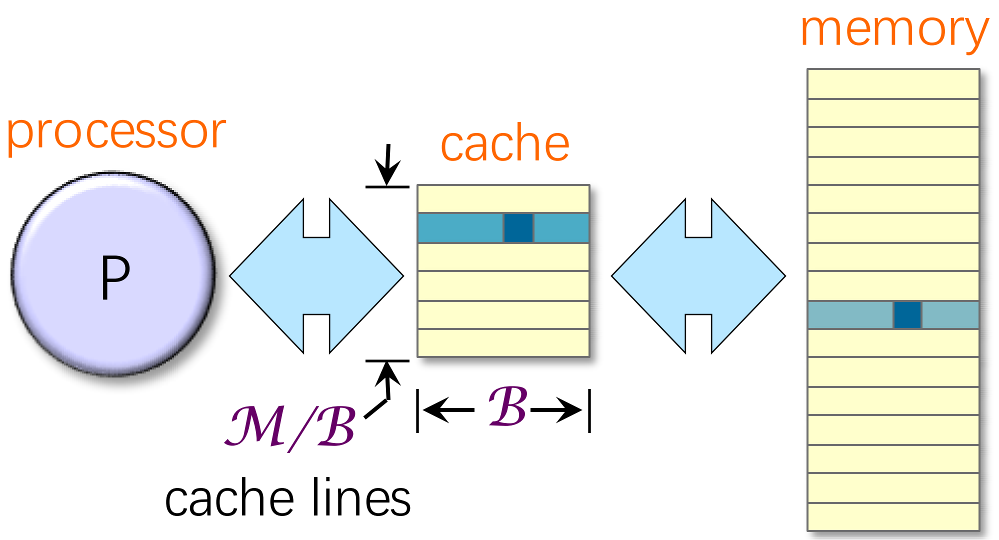
Using parallel loops gets us almost **18×** speedup on **18** cores!
(Disclaimer: Not all code is so easy to parallelize effectively.)

*Why are we still getting less than **5%** of peak?*

Hardware Caches, Revisited

- [KEY IDEA] **Reuse** data in the cache as much as possible
  - Cache **misses** are slow, and cache **hits** are fast
  - Try to make the most of the cache by **reusing data** that's already there

- **CACHE CAPACITY**
  - One Row of a matrix = 4096*8bytes = 32KB
  - L1D cache = 32KB → 1 row of one matrix
  - L2 cache = 256KB → ~8 rows



D&C Matrix Multiplication

[Key Idea] For matrix multiplication, a recursive, parallel, divide-and-conquer algorithm uses caches almost optimally.

$$\begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix} \cdot \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{bmatrix}$$

Idea: Divide the matrices into $(n/2)\times(n/2)$ submatrices.

D&C Matrix Multiplication

[KEY IDEA] For matrix multiplication, a recursive, parallel, divide-and-conquer algorithm uses caches almost optimally.

$$\begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix} \cdot \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{bmatrix}$$

$$= \begin{bmatrix} A_{00}B_{00} & A_{00}B_{01} \\ A_{10}B_{00} & A_{10}B_{01} \end{bmatrix} + \begin{bmatrix} A_{01}B_{10} & A_{01}B_{11} \\ A_{11}B_{10} & A_{11}B_{11} \end{bmatrix}$$

1. Compute C_{00} += $A_{00}B_{00}$; C_{01} += $A_{00}B_{01}$; C_{10} += $A_{10}B_{00}$; and C_{11} += $A_{10}B_{01}$ recursively in parallel.
2. Compute C_{00} += $A_{01}B_{10}$; C_{01} += $A_{01}B_{11}$; C_{10} += $A_{11}B_{10}$; and C_{11} += $A_{11}B_{11}$ recursively in parallel.

Recursive Parallel Matrix Multiply

```
void mm_dac(double *restrict C, int n_C,
            double *restrict A, int n_A,
            double *restrict B, int n_B,
            int n)
{ // C += A * B
  assert((n & (-n)) == n);
  if (n <= 1) {
    *C += *A * *B;
  } else {
#define X(M,r,c) (M + (r*(n_ ## M) + c)*(n/2))
    cilk_spawn mm_dac(X(C,0,0), n_C, X(A,0,0), n_A, X(B,0,0), n_B, n/2);
    cilk_spawn mm_dac(X(C,0,1), n_C, X(A,0,0), n_A, X(B,0,1), n_B, n/2);
    cilk_spawn mm_dac(X(C,1,0), n_C, X(A,1,0), n_A, X(B,0,0), n_B, n/2);
               mm_dac(X(C,1,1), n_C, X(A,1,0), n_A, X(B,0,1), n_B, n/2);
    cilk_sync;
    cilk_spawn mm_dac(X(C,0,0), n_C, X(A,0,1), n_A, X(B,1,0), n_B, n/2);
    cilk_spawn mm_dac(X(C,0,1), n_C, X(A,0,1), n_A, X(B,1,1), n_B, n/2);
    cilk_spawn mm_dac(X(C,1,0), n_C, X(A,1,1), n_A, X(B,1,0), n_B, n/2);
               mm_dac(X(C,1,1), n_C, X(A,1,1), n_A, X(B,1,1), n_B, n/2);
    cilk_sync;
  }
}
```

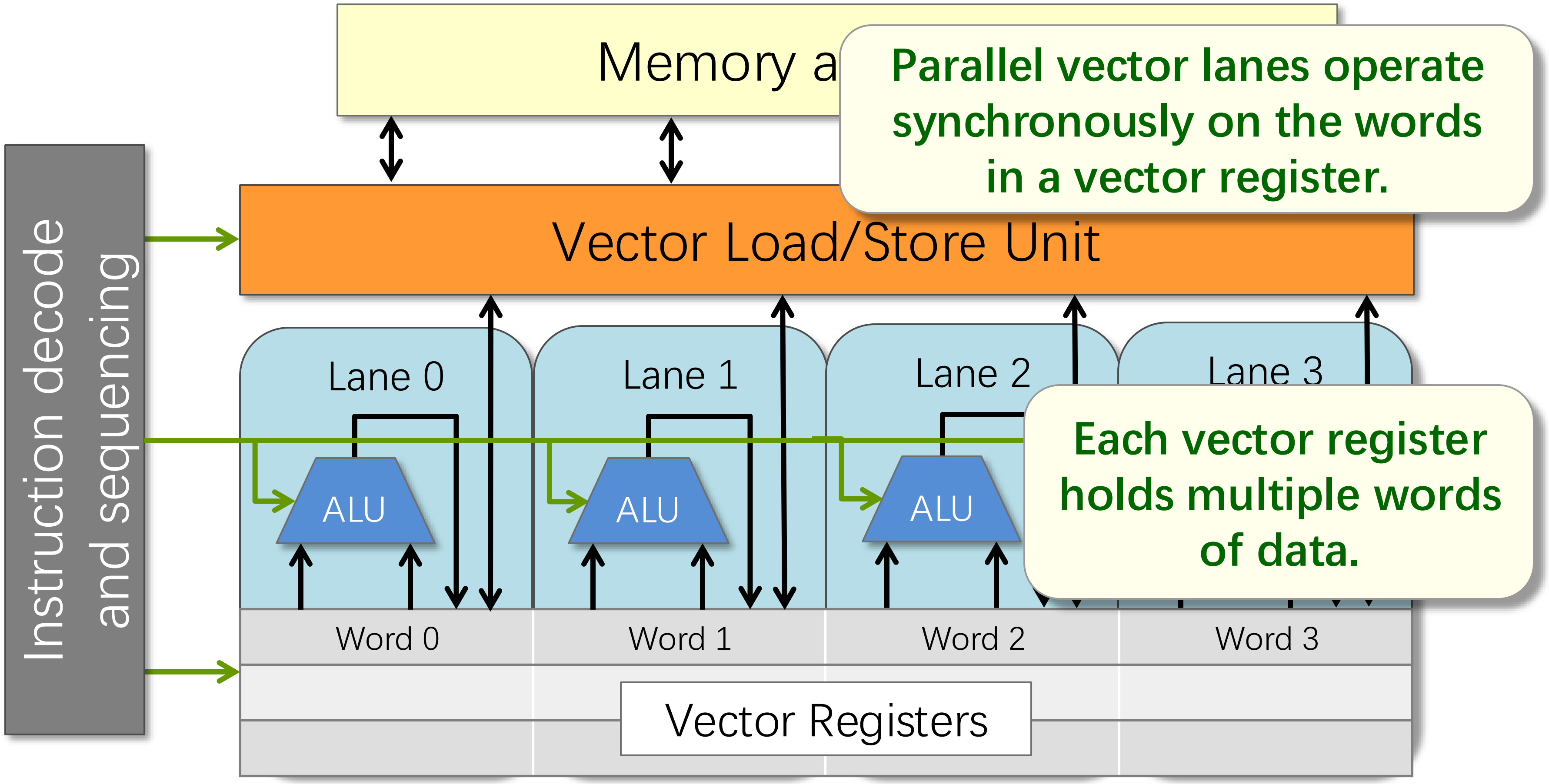
Version 7: Parallel Divide-and-Conquer

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093
5	+ optimization flags	54.63	3.25	385	2.516	0.301
6	Parallel loops	3.04	17.97	6,921	45.211	5.408
7	Parallel divide-and-conquer	1.30	2.35	16,197	105.722	12.646

Implementation	Cache references $\times 10^6$	LLC Cache misses $\times 10^6$	L1-d cache misses $\times 10^6$
Parallel loops	104,090	17,220	8,600
Parallel divide-and-conquer	58,230	9,407	64

Vector Hardware

Modern microprocessors incorporate **vector hardware** to process data in single-instruction stream, multiple-data stream (**SIMD**) fashion



Compiler Vectorization

- Clang/LLVM uses vector instructions automatically when compiling at optimization level `-O2` or higher

- Clang/LLVM can be induced to produce a **vectorization report** as follows:

```
$ clang -O3 -std=c99 mm.c -o mm -Rpass=vector
mm.c:42:7: remark: vectorized loop (vectorization width: 2,
interleaved count: 2) [-Rpass=loop-vectorize]
      for (int j = 0; j < n; ++j) {
      ^
```

Vectorization Flags

- We can use **compiler flags** to direct the compiler to use vector instructions
  - `-mavx`: Use Intel AVX vector instructions
  - `-mavx2`: Use Intel AVX2 vector instructions
  - `-mfma`: Use fused multiply-add vector instructions
  - `-march=<string>`: Use whatever instructions available on the specified architecture
  - `-march=native`: Use whatever instructions are available on the architecture of the machine doing compilation
- Due to restrictions on floating-point arithmetic, additional flags (e.g. `–ffast-math`) might be needed for vectorization flags to have an effect

Version 8: Compiler Vectorization

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
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7	Parallel divide-and-conquer	1.30	2.35	16,197	105.722	12.646
8	+ compiler vectorization	0.70	1.87	30,272	196.341	23.486

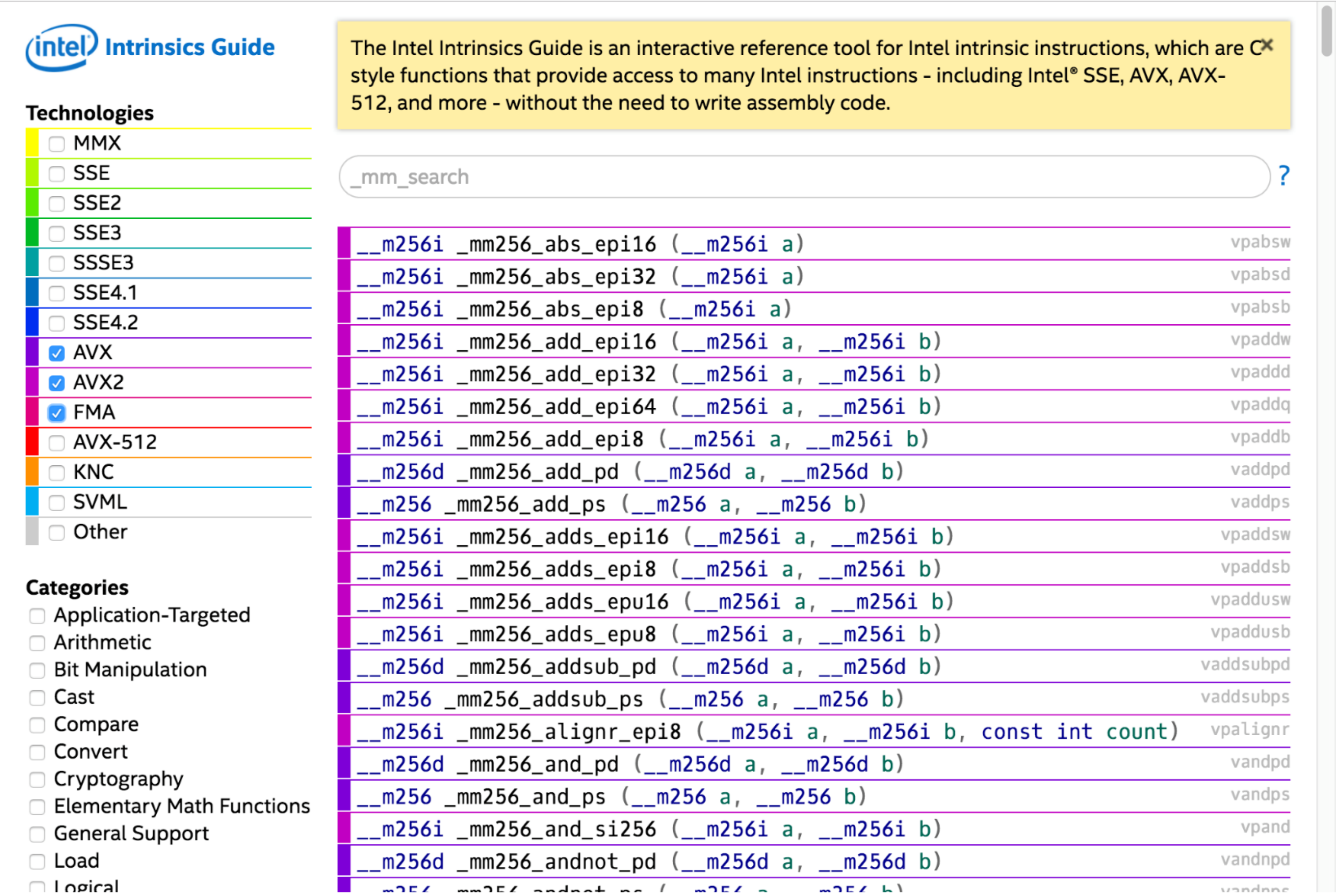
Using the flags `-march=native -ffast-math` nearly doubles the program's performance!

Can we be smarter than the compiler?

AVX Intrinsic Instructions

- Intel provides C-style functions, called **intrinsic instructions**, that provide direct access to hardware vector operations:

  https://software.intel.com/sites/landingpage/IntrinsicsGuide/

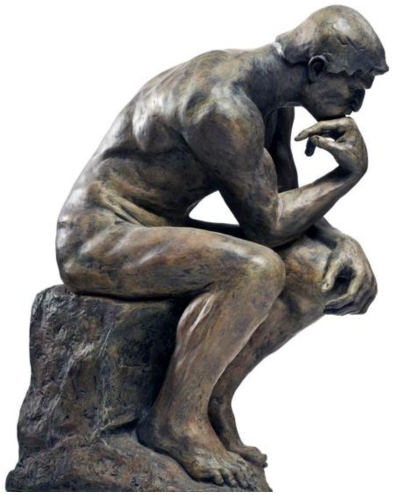


Plus More Optimizations

- We can apply several more insights and performance-engineering tricks to make this code run faster, including:
  - Preprocessing
  - Matrix transposition
  - Data alignment
  - Memory-management optimizations
  - A clever algorithm for the base case that uses AVX intrinsic instructions explicitly

Plus Performance Engineering

Think,



code,

run, run, run···

···to test and measure many different implementations

Version 9: AVX Intrinsics

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
4	+ interchange loops	177.68	6.50	118	0.774	0.093
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6	Parallel loops	3.04	17.97	6,921	45.211	5.408
7	Parallel divide-and-conquer	1.30	1.38	16,197	105.722	12.646
8	+ compiler vectorization	0.70	2.35	30,272	196.341	23.486
9	+ AVX intrinsics	0.39	1.76	53,292	352.408	41.677

Version 10: Final Reckoning

Version	Implementation	Running time (s)	Relative speedup	Absolute Speedup	GFLOPS	Percent of peak
1	Python	21041.67	1.00	1	0.006	0.001
2	Java	2387.32	8.81	9	0.058	0.007
3	C	1155.77	2.07	18	0.118	0.014
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6	Parallel loops	3.04	17.97	6,921	45.211	5.408
7	Parallel divide-and-conquer	1.30	1.38	16,197	105.722	12.646
8	+ compiler vectorization	0.70	1.87	30,272	196.341	23.486
9	+ AVX intrinsics	0.39	1.76	**53,292**	352.408	41.677
10	Intel MKL	0.41	0.97	51,497	335.217	40.098

Our Version 9 is competitive with Intel's professionally engineered Math Kernel Library (MKL)!

Performance Engineering

53,292x speedup

Galapagos
Tortoise
0.5 k/h

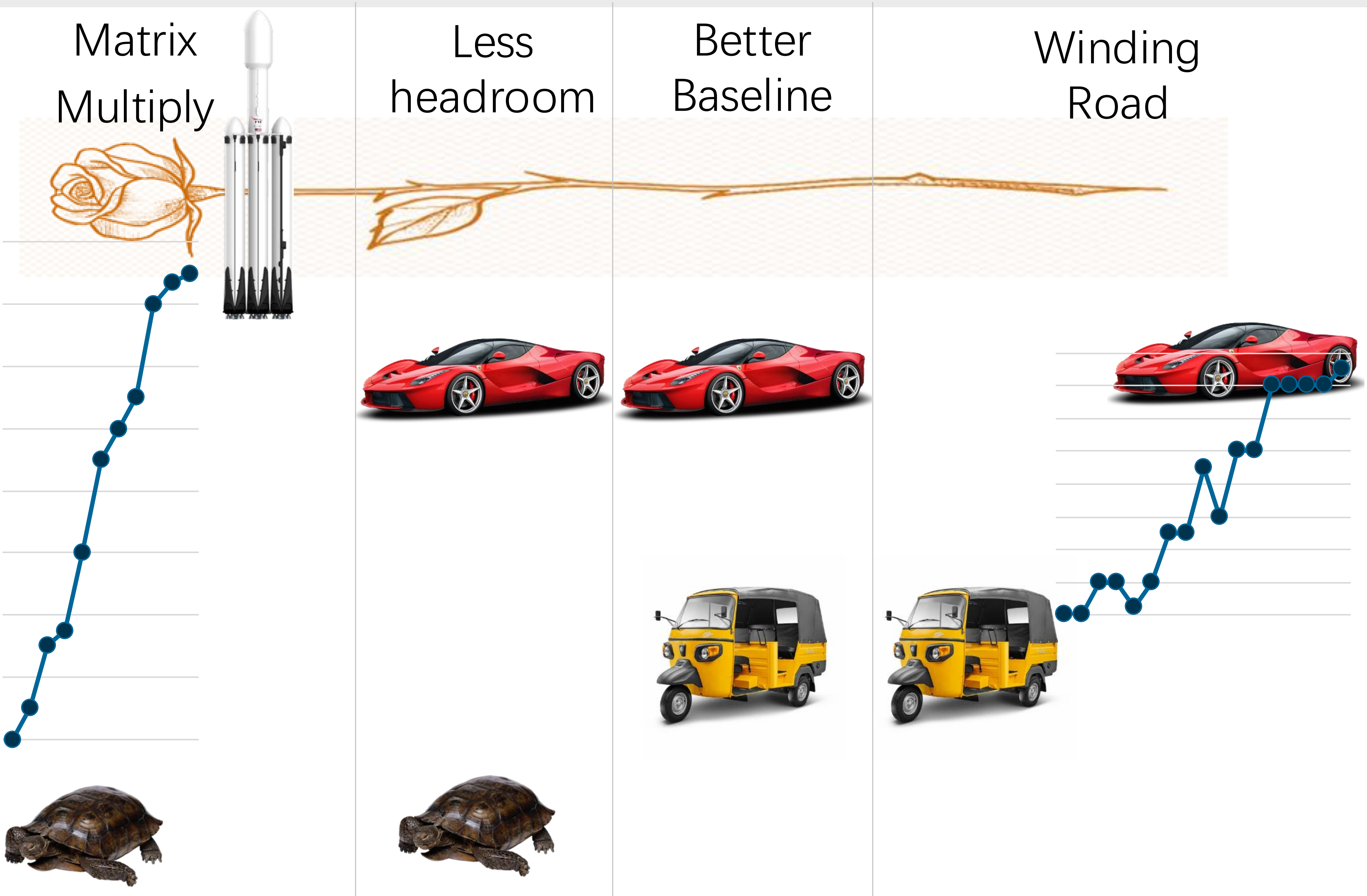
Performance Engineering

Escape
Velocity
11 k/s

53,292×

Galapagos
Tortoise
0.5 k/h

Disclaimer

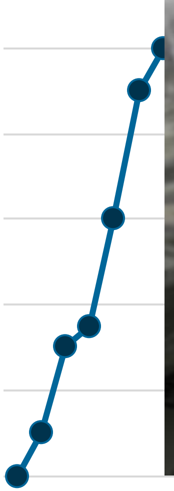


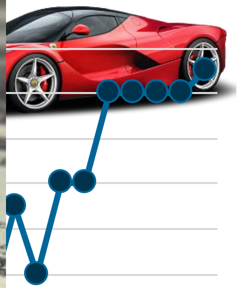
Matrix Multiply
Less headroom
Better Baseline
Winding Road

Disclaimer

Ma

Mu





- Matrix Multiplication is an exception
- But this class will teach you how to print the currency of performance all by yourself

Software Performance Engineering

Course Website

software-performance-engineering.github.io/

Xuhao Chen

Tuesday, August 26, 2025